Isospin mode splitting and mixing in asymmetric nuclear matter

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We estimate exclusive density and asymmetry parameter dependent dispersion relations of various charged states of pions in asymmetric nuclear matter. The possibility of matter induced mixing of π^0 with η is clearly exposed with the further mass modification of π^0 meson due to mixing. Asymmetry driven mass splitting and mixing amplitude are of the same order as the corresponding values in vacuum. Closed form analytic results for the mass shifts and dispersion relations with and without mixing are presented. Furthermore, we discuss the sensitivity of our results on the scalar mean field within the framework of Quantum Hadrodynamics.

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I. INTRODUCTION

It is well known that the particle dispersion changes in matter due to scatterings with the medium constituents. This is characterized by the density(ρ_b) dependent self-energy ($\Sigma(q_0, |\mathbf{q}|)$) of the propagating particle. Clearly it depends on the energy (q_0) and three momentum ($|\mathbf{q}|$) of the particle. At low density $\Sigma(q_0, |\mathbf{q}|)$ can be calculated by multiplying the forward scattering amplitude with the density, which, however fails at higher density where multiple scattering becomes important [1]. To incorporate these higher order effects one calculates full self-energy by evaluating loops at various orders. The real and imaginary parts of $\Sigma(q_0, |\mathbf{q}|)$ determine the in-medium mass and the decay width of the particle. The medium modified dispersion relations characterizing the collective excitations are obtained from the zeros of the inverse propagator involving the density dependent self-energies.

In the present paper we plan to investigate the pion propagation in asymmetric nuclear matter (ANM). Such investigations are important to understand the pion(π)-nucleon(N) dynamics at finite density. Medium modifies the pion masses. Such mass shifts in nuclear matter can be used to calculate the pion-nucleus optical potential [2,3] which are different for different charged states. Furthermore, in-medium pion dispersion relations also determine the low mass dilepton yields in relativistic heavy ion collisions which is a subject of contemporary debate [4,5]. With this motivation, we here focus on the in-medium properties of pions in neutron rich matter. This is in sharp contrast with most of the previous calculations that deal with symmetric nuclear matter (SNM) [6,7]. The possibility of asymmetry driven π-η mixing, as explored here, has not been addressed before. Such mixing also modifies the pion mass in ANM. It should be noted that such mixing of different isospin states and the mass splitting of various charged states in ANM is a generic effect and therefore interesting in itself. Furthermore, in the present study we go beyond Fermi gas model and incorporate N-N interactions within the framework of Quantum Hadrodynamics (QHD) [8].

Pion propagation in ANM is qualitatively different from SNM. This, as we show, is due to the possible splitting and mixing of the collective modes. Both the splitting of isospin multiplets and the coupling of the isoscalar and isovector oscillations are related to the ground state induced isospin symmetry breaking [9–12]. Clearly charged and neutral pion masses will be non-degenerate once the density dependent dressings of the pion propagators in ANM are considered. Such splitting may also be observed in vacuum due to virtual nucleon-antinucleon excitations if the mass(M) splitting of neutron(n) and proton(p) are taken into account (or at the quark level u-d mass difference). In addition under this situation π^0 (pure isospin eigen state) may contain admixture of η meson. Such π-η meson mixing in vacuum due to the n-p mass difference has been considered in ref. [13]. But the mechanism we propose here for such splitting and mixing is generically different. Here, as we shall see, it is driven by the difference of the proton and neutron Fermi momentum ($k^F_{p,n}$), *i.e.* $k^F_p \neq k^F_n$. This possibility, in the context of ρ-ω mixing, was first suggested by one of the present authors [11]. Moreover, to focus exclusively on the density dependent effect we neglect explicit symmetry breaking and take $M_p = M_n$.

II. FORMALISM

To describe pion-nucleon interaction we consider pseudovector coupling :

$$\mathcal{L}^{PV}_{int} = -\frac{f_\pi}{m_\pi}\bar{\Psi}_N\gamma_5\gamma_\mu\partial^\mu\left(\vec{\tau}\cdot\vec{\Phi}_\pi\right)\Psi_N \tag{1}$$

Here $\frac{f_\pi}{m_\pi} = \frac{g_\pi}{2M}$ and $\frac{g_\pi^2}{4\pi} = 12.6$. In Eq.1 Ψ_N and Φ_π are the nucleon and pion fields respectively and $\vec{\tau}$ is the isospin operator. The η-NN interaction Lagrangian for pseudovector coupling can be found from Eq.1 with the following substitution : $\frac{f_\pi}{m_\pi} \to \frac{f_\eta}{m_\eta}$ and $\vec{\tau}\cdot\vec{\Phi}_\pi \to \Phi_\eta$. Here $\frac{f_\eta}{m_\eta} = \frac{g_\eta}{2M}$ and $\frac{g_\eta^2}{4\pi} = 5.5$.

Another essential ingredient to calculate in-medium self-energy of the π/η meson is the in-medium nucleon Green's function. In nuclear matter usual vacuum is replaced by the occupied Fermi sea which forbids on mass-shell nucleon propagation below the Fermi momentum because of the Pauli blocking. The relativistic nucleon propagator in this case is given by $G(k) = G^F(k) + G^D(k)$, where the superscript F and D denotes the free and dense part respectively. Explicitly they are given by

$$G^F(k) = \frac{\not{k} + M^*}{k^2 - M^{*2} + i\zeta}, \quad G^D(k) = \frac{i\pi(\not{k} + M^*)}{\epsilon^*}\delta(k_0 - \epsilon^*)\theta(k^F - |\mathbf{k}|) \tag{2}$$

Here k^F is the Fermi momentum and M^* denotes medium modified nucleon mass. The energy of nucleon is $\epsilon^* = \sqrt{M^{*2} + \mathbf{k}^2}$. We, from now onwards, use k_p and k_n to denote the proton and neutron Fermi momentum respectivel. In QHD the effective nuclear mass is determined from the following self-consistent condition [8].

$$M^* = M - \frac{g_\sigma^2}{M_\sigma^2}(\rho_p^s + \rho_n^s), \tag{3}$$

where ρ_i^s $(i = p, n)$ represent scalar densities given by

$$\rho_i^s = \frac{M^*}{2\pi^2}\left[\epsilon_i^* k_i - M^{*2}\ln\left(\frac{\epsilon_i^* + k_i}{M^*}\right)\right] \tag{4}$$

The general formula for calculating self-energy is given below.

$$\Sigma_{\pi\pi}(q) = -\, i \int \frac{d^4k}{(2\pi)^4} Tr\left[\{\, i\Gamma(q)\}\; iG_{p(n)}(k+q)\{\, i\Gamma(-q)\}\; iG_{p(n)}(k)\right] \tag{5}$$

Where $G_{p(n)}$ denotes the proton (neutron) propagator and $\Gamma(q) = -\,\frac{f_\pi}{m_\pi}\gamma_5 i\not{q}$ is the vertex factor. With Eq.2 and Eq.5 the pion self-energy due to scattering from Fermi sphere takes the following form :

$$\Sigma_{\pi\pi}(q) = (-i)\int \frac{d^4k}{(2\pi)^4}\left(\frac{f_\pi}{m_\pi}\right)^2 \mathbf{T} \tag{6}$$

Here $\mathbf{T}$ is the trace factor. For π^0

$$\mathbf{T} = Tr[\gamma_5\not{q}G_p^F(k+q)\gamma_5\not{q}G_p^D(k) + \gamma_5\not{q}G_p^D(k+q)\gamma_5\not{q}G_p^F(k)] + [p \to n] \tag{7}$$

But for $\pi^{+(-)}$, f_π is to be replaced by $\sqrt{2}f_\pi$ ($\sqrt{2}$ is the isospin factor) and

$$\mathbf{T} = Tr[\gamma_5\not{q}G_{p(n)}^F(k+q)\gamma_5\not{q}G_{n(p)}^D(k) + \gamma_5\not{q}G_{p(n)}^D(k+q)\gamma_5\not{q}G_{n(p)}^F(k)] \tag{8}$$

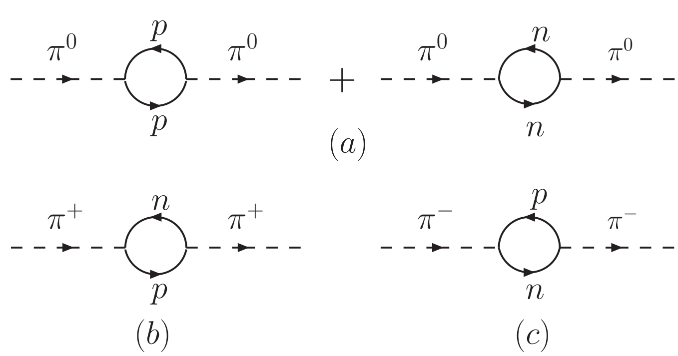

FIG. 1. (a) represents the one-loop self-energy diagram for π^0, (b) and (c) represents the same for π^+ and π^- respectively.

Now the self-eneries for π^0 and $\pi^\pm$ can be determined by evaluating the Fig. 1(a), 1(b) and 1(c) respectively. Using Eqs.7 and 8 in Eq.6 we get,

$$\Sigma^0_{\pi\pi}(q) = -\ 8\ \left(\frac{f_\pi}{m_\pi}\right)^2 \int \frac{d^3k}{(2\pi)^2\epsilon^*_k}\ \mathbf{A} \tag{9}$$

and

$$\Sigma^\pm_{\pi\pi}(q) = -\ 8\ \left(\frac{f_\pi}{m_\pi}\right)^2 \int \frac{d^3k}{(2\pi)^2\epsilon^*_k}\ [\mathbf{A} \mp \mathbf{B}] \tag{10}$$

Where

$$\mathbf{A} = \left[\frac{M^{*2}q^4}{q^4 - 4(k.q)^2}\right](\theta_p + \theta_n) \tag{11}$$

and

$$\mathbf{B} = \frac{1}{2}\left[1 + \frac{M^{*2}q^2}{q^4 - 4(k.q)^2}\right](k.q)\,(\theta_p - \theta_n) \tag{12}$$

with $\theta_{p,n} = \theta(k_{p,n} - |\mathbf{k}|)$. The superscripts on $\Sigma^{0,\pm}$ represent self-energies for various charged states of pion. We restrict ourselves in the long wave length limit *i.e.* when the pion momentum ($|\mathbf{q}|$) is small compared to the Fermi momentum($k^F_{p,n}$) of the system where the many body effects manifest strongly. In this regime the concept of individual scattering breaks down and particle propagation can be understood in terms of the low momentum collective excitations of the system. In the short wave-length limit, *viz.* at distance scale much small compared to the inter particle separation ($1/k^F_{p,n}$), one observes that the particle dispersion approaches to that of the free propagation. The recognition of the special role played by the distant collisions in determining the collective excitations permits analytical solutions of the dispersion relations [14,15].

III. RESULTS

Now we present main results of our calculations. First we evaluate the pion self-energy restricting ourselves to the long wave length limit *i.e.* we consider excitations near the Fermi surface due to scattering. In effect, we assume that the loop momentum $k \sim k^F$ and $q << k^F$. This permits us to neglect q^4 compared to the term $4(k.q)^2$ from the denominator of both $\mathbf{A}$ and $\mathbf{B}$ in Eqs.11 and 12. Explicitly, after performing the integration we get,

$$\Sigma^0_{\pi\pi}(q_0, |\mathbf{q}|) = \frac{1}{2}\left(\frac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\ln\left|\frac{1+v_p}{1-v_p}\right| - c_0 \ln\left|\frac{1+\frac{v_p}{c_0}}{1-\frac{v_p}{c_0}}\right|\right] + [p \to n] \tag{13}$$

and

$$\Sigma^\pm_{\pi\pi}(q_0, |\mathbf{q}|) = \Sigma^0_{\pi\pi}(q_0, |\mathbf{q}|) \mp \delta\Sigma_{\pi\pi}(q_0, |\mathbf{q}|) \tag{14}$$

where

$$\delta\Sigma_{\pi\pi}(q_0, |\mathbf{q}|) = \left(\frac{f_\pi}{m_\pi \pi}\right)^2 \left[\frac{2}{3}k_p^3 q_0 - \frac{M^{*2}q^2}{|\mathbf{q}|}\left(\epsilon^*_p \ln\left|\frac{1+\frac{v_p}{c_0}}{1-\frac{v_p}{c_0}}\right| - \frac{2M^{*2}}{\sqrt{c_0^2-1}}\tan^{-1}\frac{k_p\sqrt{c_0^2-1}}{c_0 M^*}\right)\right] - [p \to n] \tag{15}$$

where $c_0 = q_0/|\mathbf{q}|$, $v_{p,n} = k_{p,n}/\epsilon^*_{p,n}$ and $\epsilon^*_{p,n} = \sqrt{k^2_{p,n} + M^{*2}}$. In the limit of $|\mathbf{q}| = 0$, Eq.13 and Eq.14 reduces to the following form

$$\Sigma^0_{\pi\pi}(q_0, 0, k_{p,n}) = \Omega^2_{\pi\pi}q_0^2, \qquad \Sigma^\pm_{\pi\pi}(q_0, 0, k_{p,n}) = \Omega^2_{\pi\pi}q_0^2 \pm m_\pi \delta\Omega^2_{\pi\pi}q_0, \tag{16}$$

where

$$\Omega^2_{\pi\pi} = \left(\frac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\frac{1}{3}\left(\frac{k_p^3}{\epsilon_p^{*3}} + \frac{k_n^3}{\epsilon_n^{*3}}\right) + \frac{1}{5}\left(\frac{k_p^5}{\epsilon_p^{*5}} + \frac{k_n^5}{\epsilon_n^{*5}}\right)\right] \tag{17}$$

$$\delta\Omega^2_{\pi\pi} = \frac{1}{m_\pi}\left(\frac{f_\pi M^*}{m_\pi \pi}\right)^2 \left[\frac{2}{5}\left(\frac{k_p^5}{M^{*4}} - \frac{k_n^5}{M^{*4}}\right)\right] \tag{18}$$

It is to be noted that both $\Omega^2_{\pi\pi}$ and $\delta\Omega^2_{\pi\pi}$ are dimensionless quantities. From Eq. 16 and 18, it is evident that the asymmetry driven mass splitting is an order $O(k^{F5}/M^{*5})$ effect therefore does not appear in calculations at leading order in density.

A. Isospin mode splitting of pion

Once determined, the self-energies can be used to solve the Dyson-Schwinger equation to derive the dispersion relations, *viz.*

$$q_0^2 - \mathbf{q}^2 - m^2_{\pi^{0\pm}} - \Sigma^{0,\pm}_{\pi\pi}(q_0, |\mathbf{q}|) = 0 \tag{19}$$

In ANM $\Sigma^0 \neq \Sigma^+ \neq \Sigma^-$, hence the pion effective masses will split in matter. The origin of mass splitting can be understood by considering $\pi^{0,\pm}$ scatterings from the Fermi surface in different channels. This would correspond to various cuts of Fig. 1 due to the G^D term in Eq.2. It is clear that for $\pi^\pm$ scattering we have contributions both from the direct (s) and exchange (u) channels. However for the π^+ in the s channel neutron (n) density contributes while the u channel involves proton (p) Fermi sphere. For the π^- self-energy the role of p and n gets reversed. While for π^0 self-energy, both for the s and u channel, the sum of p and n densities appear.

In the static limit ($|\mathbf{q}| = 0$) following expressions for the pion effective masses (q_0) are found.

$$m^{*2}_{\pi^0} \simeq \frac{m^2_{\pi^0}}{1 - \Omega^2_{\pi\pi}} \qquad m^{*2}_{\pi^\pm} \simeq \frac{m^2_{\pi^\pm}}{1 - (\Omega^2_{\pi\pi} \pm \delta\Omega^2_{\pi\pi})} \tag{20}$$

The approximate mass-shifts of $\pi^{0\pm}$ are :

$$\Delta m_{\pi^0} \simeq \frac{m_{\pi^0}}{2}\Omega^2_{\pi\pi}, \quad \Delta m_{\pi^\pm} \simeq \frac{m_{\pi^\pm}}{2}\left[\Omega^2_{\pi\pi} \pm \delta\Omega^2_{\pi\pi}\right] \tag{21}$$

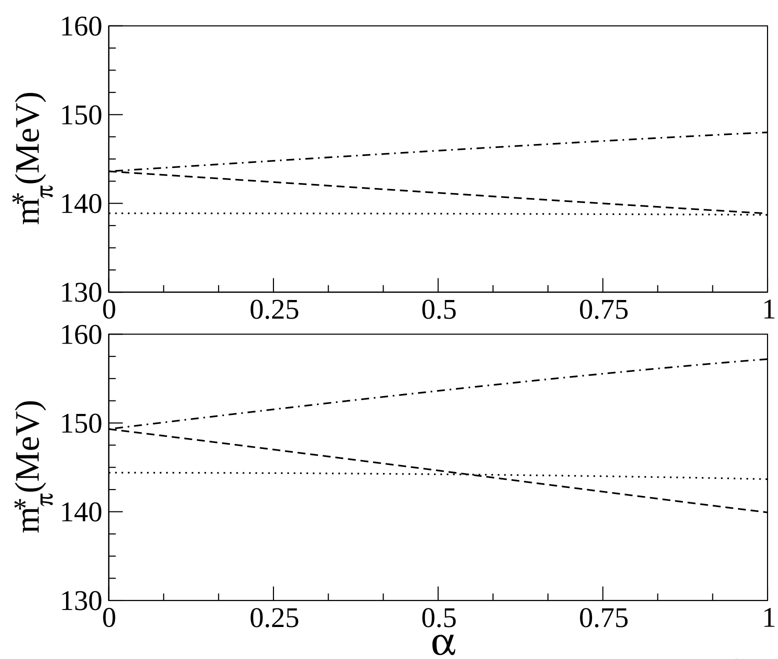

FIG. 2. The dashed, dotted and dashed-dotted lines represent the effective masses of π^+,π^0,π^- respectively. Upper panel represents effective masses in the free Fermi gas approximation and lower panel represents the same with interaction via the scalar mean field.

From the above equation we find that the splitting is caused by the term $\delta\Omega^2_{\pi\pi}$ which for SNM ($k_p = k_n$) vanishes and $\pi^{\pm}$ masses, as expected, become degenerate.

Furthermore, it should be noted that this splitting is an higher order effect $\sim k^5_{p,n}/\epsilon^5_{p,n}$ and therefore at the linear density level ($k^3_{p,n}/\epsilon^3_{p,n}$) all the pion masses will remain degenerate, if the free pion masses are assumed to be so. We have seen that all higher order density corrections to the self-energy vanish in the static limit $|\mathbf{q}| = 0$. Numerical estimation for the effective pion masses are shown in Fig. 2 as a function of the asymmetry parameter $\alpha = (\rho_n - \rho_p)/\rho_b$ at normal nuclear matter density. Evidently in neutron rich matter the π^- meson shows more repulsive behaviour than π^0 and π^+ meson which follows from Eq. 18. Moreover the π^0 mass is rather insensitive to α. We find that all the masses receive an overall density dependent positive corrections consistent with Eq. 17 and 18. In the upper panel we show the results corresponding to the free Fermi gas model, while in the lower panel effect of the interacting ground state is taken via Relativistic Hartree Approximation (RHA) [8].

The mean field, as we see from the lower panel of Fig. 2, enhances the mass splitting. It is observed that although the mass splitting contribute at $O(k^5_{p,n}/\epsilon^5_{p,n})$, the correction is large compared to the vacuum mass difference (Fig. 2). At normal nuclear matter density $\Delta m_{\pi^+} = -1.15(3.39)$ MeV, $\Delta m_{\pi^0} = 4.85(5.01)$ MeV and $\Delta m_{\pi^-} = 11.19(6.98)$ MeV at $\alpha = 0.7(0.2)$ with mean field ($M^*_p = M^*_n = 685.0$ MeV).

The dispersion relation for $\pi^{0\pm}$ is

$$(q_0^2)_{\pi^{0\pm}} \simeq m^{*2}_{\pi^{0\pm}} + \gamma_{\pi\pi}\mathbf{q}^2 + \left(\frac{\gamma^2_{\pi\pi}}{4} + \alpha_{\pi\pi}\right)\frac{\mathbf{q}^4}{m^{*2}_{\pi^{0\pm}}} \tag{22}$$

Effective pion masses $m^*_{\pi^{0,\pm}}$ are given by Eq. 20. Here

$$\gamma_{\pi\pi} = 1 - \chi_{\pi\pi} + \beta_{\pi\pi}, \qquad \chi_{\pi\pi} = \frac{\Omega^2_{\pi\pi}}{1 - \Omega^2_{\pi\pi}} \tag{23}$$

$$\alpha_{\pi\pi} = \left(\frac{f_\pi M^*}{m_\pi \pi}\right)^2 \frac{1}{3(1-\Omega^2_{\pi\pi})}\left(\frac{k_p^3}{\epsilon_p^{*3}} + \frac{k_n^3}{\epsilon_n^{*3}}\right) \tag{24}$$

$$\beta_{\pi\pi} = \left(\frac{f_\pi M^*}{m_\pi \pi}\right)^2 \frac{1}{5(1-\Omega^2_{\pi\pi})}\left(\frac{k_p^5}{\epsilon_p^{*5}} + \frac{k_n^5}{\epsilon_n^{*5}}\right) \tag{25}$$

B. Matter induced π-η mixing

Another consequence of $n \leftrightarrow p$ asymmetric ground state, as mentioned before, is the isospin mode mixing. This is an exclusive density dependent effect. To see this we recall that

$$\mathcal{L}^{PV}_{\pi^0} = -\frac{f_\pi}{m_\pi}\left[\bar{\Psi}_p\gamma_5\gamma_\mu\partial^\mu\Phi_{\pi^0}\Psi_p - \bar{\Psi}_n\gamma_5\gamma_\mu\partial^\mu\Phi_\pi\Psi_n\right] \tag{26}$$

$$\mathcal{L}^{PV}_{\eta} = -\frac{f_\eta}{m_\eta}\left[\bar{\Psi}_p\gamma_5\gamma_\mu\partial^\mu\Phi_{\eta}\Psi_p + \bar{\Psi}_n\gamma_5\gamma_\mu\partial^\mu\Phi_\eta\Psi_n\right] \tag{27}$$

The π^0 couples to p and n with opposite sign (Eq.26) while η couples with the same sign (Eq.27). This brings in a relative sign between the proton and neutron loop as shown in Fig 4 which forces mixing amplitude to vanish in SNM ($k_p = k_n$) and become non-zero in ANM ($k_p \neq k_n$). The π^0-η mixing amplitude in the static limit is evaluated to be

$$\Sigma^0_{\pi\eta}(q_0, 0) = \Omega^2_{\pi\eta} q_0^2 \tag{28}$$

where

$$\Omega^2_{\pi\eta} = \left(\frac{f_\pi M^*}{m_\pi \pi}\right)\left(\frac{f_\eta M^*}{m_\eta \pi}\right)\left[\frac{1}{3}\left(\frac{k_p^3}{\epsilon_p^{*3}} - \frac{k_n^3}{\epsilon_n^{*3}}\right) + \frac{1}{5}\left(\frac{k_p^5}{\epsilon_p^{*5}} - \frac{k_n^5}{\epsilon_n^{*5}}\right)\right] \tag{29}$$

Note the difference of sign in Eq.29 and Eq.17. Clearly $\Omega^2_{\pi\eta}$ is non-zero in ANM and vanishes for $\epsilon^*_p = \epsilon^*_n$.

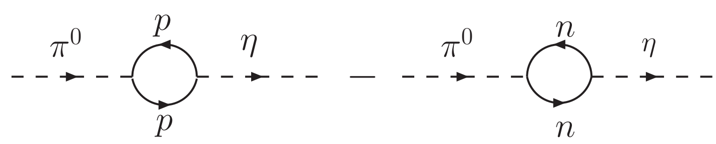

FIG. 3. This is the one-loop diagram for π^0-η mixing .

In presence of mixing the pion and eta propagation gets coupled with each other and can be represented by 2×2 matrix [8,11]

$$\begin{pmatrix} 1 - \frac{\Sigma^0_{\pi\pi}}{q_0^2 - m_\pi^2} & \frac{\Sigma^0_{\pi\eta}}{q_0^2 - m_\pi^2} \\ \frac{\Sigma^0_{\pi\eta}}{q_0^2 - m_\eta^2} & 1 - \frac{\Sigma^0_{\eta\eta}}{q_0^2 - m_\eta^2} \end{pmatrix}. \tag{30}$$

Shifted masses are obtained by solving the following equation.

$$(q_0^2 - m_\pi^2 - \Sigma^0_{\pi\pi}(q_0, 0))(q_0^2 - m_\eta^2 - \Sigma^0_{\eta\eta}(q_0, 0)) - (\Sigma^0_{\pi\eta}(q_0, 0))^2 = 0 \tag{31}$$

The Eq.31 can be written as

$$q_0^4(1 - \Delta^2) - q_0^2(m^{*2}_{\pi^0} + m^{*2}_\eta) + m^{*2}_{\pi^0} m^{*2}_\eta = 0 \tag{32}$$

Where $m^*_{\pi^0}$ and m^*_η are the effective masses of π^0 and η . Here

$$m^{*2}_\eta = \frac{m_\eta^2}{1 - \Omega^2_{\eta\eta}} \qquad \Delta^2 = \frac{\Omega^2_{\pi\eta}}{1 - \Omega^2_{\pi\pi}} \; \frac{\Omega^2_{\pi\eta}}{1 - \Omega^2_{\eta\eta}} \tag{33}$$

Solving Eq.32 we get the modified effective masses of π^0 and η with mixing :

$$\tilde{m}_{\pi^0} \simeq m^*_{\pi^0}(1 + \frac{1}{2}\Delta^2) - \frac{m^*_\eta}{2} \left[\frac{m^*_{\pi^0} m^*_\eta}{m^{*2}_\eta - m^{*2}_\pi} \right] \Delta^2 \tag{34}$$

$$\tilde{m}_\eta \simeq m^*_\eta(1 + \frac{1}{2}\Delta^2) + \frac{m^*_{\pi^0}}{2} \left[\frac{m^*_{\pi^0} m^*_\eta}{m^{*2}_\eta - m^{*2}_\pi} \right] \Delta^2 \tag{35}$$

Note that for SNM, $\Delta^2 = 0$ yielding medium modified masses as given in Eq.20.

Finally we present pionic dispersion relations in asymmetric nuclear matter with the possible π^0-η mixing :

$$(q_0^2)_{\pi^0} \simeq \tilde{m}^2_{\pi^0} - \left[\frac{m^{*2}_{\pi^0}(\gamma_{\pi\pi} + \gamma_{\eta\eta} - \delta^2)}{(1 - \Delta^2)(m^{*2}_\eta - m^{*2}_{\pi^0})} - \frac{(\gamma_{\pi\pi} m^{*2}_\eta + \gamma_{\eta\eta} m^{*2}_{\pi^0})}{(m^{*2}_\eta - m^{*2}_{\pi^0})} \right] \mathbf{q}^2 \tag{36}$$

$$(q_0^2)_\eta \simeq \tilde{m}^2_\eta + \left[\frac{m^{*2}_\eta(\gamma_{\pi\pi} + \gamma_{\eta\eta} - \delta^2)}{(1 - \Delta^2)(m^{*2}_\eta - m^{*2}_{\pi^0})} - \frac{(\gamma_{\pi\pi} m^{*2}_\eta + \gamma_{\eta\eta} m^{*2}_{\pi^0})}{(m^{*2}_\eta - m^{*2}_{\pi^0})} \right] \mathbf{q}^2 \tag{37}$$

Where

$$\delta^2 = 4 \; \frac{\Omega^2_{\pi\eta}}{1 - \Omega^2_{\pi\pi}} \; \left[\frac{\Omega^2_{\pi\eta} - \frac{1}{2}\beta_{\pi\eta}}{1 - \Omega^2_{\eta\eta}} \right] \tag{38}$$

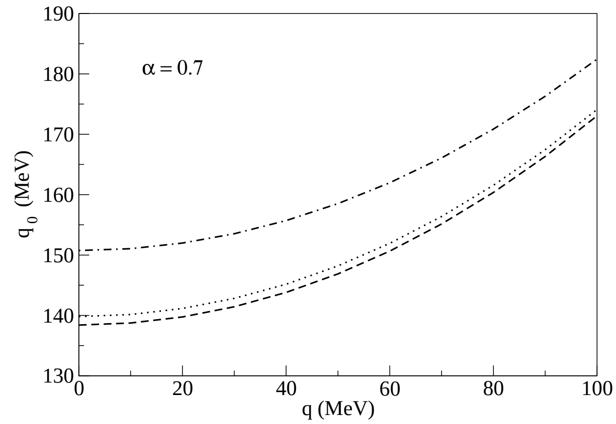

FIG. 4. The dashed, dotted and dashed-dotted curves represents the dispersion characteristics for π^+,π^0,π^- respectively at the asymmetry parameter $\alpha = 0.7$.

In SNM Δ^2 and δ^2 vanish and Eq.36 gives the same dispersion for π^0 as Eq.22. In Fig. 4 dispersion relations for various charged states are depicted. The pure density dependent mixing at the η pole is estimated to be $\Sigma_{\pi\eta}^{matt} = -1217.475$ MeV^2, $\Sigma_{\pi\eta}^{matt} = -1661.11$ MeV^2, at $\alpha = 0.2$ and $\alpha = 0.3$ respectively at normal nuclear matter density with coupling parameters same as that of [13] which shows that even at normal nuclear matter density the mixing amplitudes are of the same order as that of the vacuum mixing $\Sigma_{\pi\eta}^{vac} = -4200$ MeV^2 amplitude [13]. At higher density the matter induced mixing significantly increases.

IV. CONCLUSION

To conclude, in the present work we have shown the characteristic changes of pion dispersion relations in ANM with the possibility of splitting and mixing of different isospin states. A density expansion of the pion self-energy due to scattering from Fermi sphere is made to obtain analytical results. We also have considered the modification of π^0 mass due to its mixing with the η meson. Furthermore, it is found that the π^- in neutron rich matter experiences more repulsion than $\pi^{0,+}$ in agreement with the chiral perturbation theory calculation [3]. Both the mass splitting (although shown to be an $O(k_{n,p}^5/\epsilon_{n,p}^5)$ effect) and the π-η mixing amplitude for neutron rich system are comparable to their vacuum counterparts. It is found that the density dependent mixing is quite important an effect even at normal nuclear density for asymmetry in the range of $\alpha = 0.2 - 0.3$, at higher density (and/or higher asymmetry) both the mass splitting and mixing effects are dramatically higher. To focus exclusively on the density dependent effects the π-mass splitting due to *n*-*p* mass difference has purposely been neglected. Present calculation can be extended to include such effects which would modify the quantitative results only. We also have included scalar mean field to understand the effect of interacting ground state.

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